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If, in (8), $l=a+b$, $m=b+c$, $n=a+c$, we have

$$x^2+2ax+a^2=0, \text{ or } x=-a\ldots(9).$$

Consider the following special case.

Let $a=12$, $b=10$, $c=8$, $l=15$, $m=11$, $n=13$, and put the equations for the four cases in the general form,

$$Ax^2-2Bx+C=0\ldots(10),$$
$$\text{or } x=\frac{B\pm\sqrt{[B^2-AC]}}{A}.$$

Then, $C=(a^2b^2+c^2l^2)(l^2-m^2-n^2)+(b^2c^2+a^2m^2)(m^2-n^2-l^2)$
$+(a^2c^2+b^2n^2)(n^2-l^2-m^2)+c^4l^2+a^4m^2+b^4n^2+l^2m^2n^2$
$=-3099803$, the same for all cases.
$(l+m+n)(l+m-n)(l-m+n)(m-l+n)=77571$;
$cl^2(l^2-m^2-n^2)=-117000$; $am^2(m^2-n^2-l^2)=-396396$;
$bn^2(n^2-l^2-m^2)=-299130$.

Then, for Case I, $A=-69203$, $B=-730510$, and $x=2.3929$ or 18.7192.
Case II, $A=167917$, $B=-271610$, and $x=\pm2.6775$ or ±5.9125.
Case III, $A=129133$, $B=-346198$, and $x=\pm2.9041$ or ±8.2659.
Case IV, $A=241453$, $B=-112702$, and $x=\pm3.1465$ or ±4.0801.

As another example, let a, b, c, be the same as before, but let $l=30$, $m=20$, $n=25$. Then we find

$C=113822500$; $(l+m+n)(l-m+n)(l+m-n)(m-l+n)=984375$;
$cl^2(l^2-m^2-n^2)=-900000$; $am^2(m^2-n^2-l^2)=-5400000$;
$bn^2(n^2-m^2-l^2)=-4218750$.

Then, Case I, $A=-1010375$, $B=-10757950$, and $x=-4.3868$, or 25.6818.
Case II, $A=-112775$, $B=-2473250$, and $x=\pm16.6718$, or ±60.5335.
Case III, $A=-314375$, $B=-838750$, and $x=\pm16.5460$, or ±21.8820.
Case IV, $A=285625$, $B=-6898750$, $x=\pm10.5564$, and ±37.7500.

CALCULUS.

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243. Proposed by R. D. CARMICHAEL, Anniston, Ala.

The usual method for the solution of a differential equation in the form (see Cohen, *Differential Equations*, p. 22)

$$x^ry^s(my\,dx+nx\,dy)+x^\rho y^\sigma(\mu y\,dx+\nu x\,dy)=0$$

fails when (1) $n=am$, (2) $\nu=a\mu$, (3) $s-\sigma\neq a(r-\rho)$. Find the solution when the relations (1) and (2) hold. (Note that the solution desired does not depend on (3).)

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

By making the substitution indicated, we get

$$mx^r y^s (ydx+axdy)+\mu x^\rho y^\sigma (ydx+axdy)=0.$$

$\therefore ydx+axdy=0$, and $mx^r y^s+\mu x^\rho y^\sigma=0$.

$\therefore \dfrac{dx}{x}+a\dfrac{dy}{y}=0$, and, therefore, $\log x+a\log y=\log C$, or $\log(xy^a)=\log C$ or $xy^a=C$, therefore, $x=C/y^a$.

$\therefore mC^r/y^{ar-s}+\mu C^\rho/y^{a\rho-\sigma}=0$, therefore, $y^{ar-s-a\rho+\sigma}=-\dfrac{m}{\mu}C^{r-\rho}$.

$\therefore y^{a(r-\rho)-(s-\sigma)}=-\dfrac{m}{\mu}C^{r-\rho}$, and, therefore, $y=(-\dfrac{m}{\mu}C^{r-\rho})^{1/[a(r-\rho)-(s-\sigma)]}$, and $x=C(-\dfrac{m}{\mu}C^{r-\rho})^{a/[(s-\sigma)-a(r-\rho)]}$.

244. Proposed by G. B. M. ZERR, Ph. D., Professor of Mathematics in Central Manual Training School, Philadelphia, Pa.

Fine the volume common to the solids bounded by the surfaces
$x^{2/3}+y^{2/3}+z^{2/3}=a^{2/3}$ and $x^{1/3}z^{2/3}=(a^{1/3}-x^{1/3})(x^{2/3}+y^{2/3})$.

Solution by the PROPOSER.

The limits of z are $z=\left[\dfrac{a^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{1}{3}}}(x^{\frac{2}{3}}+y^{\frac{2}{3}})\right]^{\frac{3}{2}}$ to $z=(a^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}$. Eliminating z from the equations, the limits of y are $y'=[x^{\frac{1}{3}}(a^{\frac{1}{3}}-x^{\frac{1}{3}})]^{\frac{3}{2}}$ and 0; of x, 0 and a.

$$\therefore V=4\int_0^a\int_0^{y'}\left[(a^{\frac{2}{3}}-x^{\frac{2}{3}}-y^{\frac{2}{3}})^{\frac{3}{2}}-\left(\frac{a^{\frac{1}{3}}-x^{\frac{1}{3}}}{x^{\frac{1}{3}}}(x^{\frac{2}{3}}+y^{\frac{2}{3}})\right)^{\frac{3}{2}}\right]dx\,dy.$$

Let $x=u^3$, $y=v^3$, $a=b^3$. Then we get

$$V=36\int_0^b\int_0^{\sqrt{[u(b-u)]}}u^2v^2\{(b^2-u^2-v^2)^{\frac{3}{2}}-[\frac{b-u}{u}(u^2+v^2)]^{\frac{3}{2}}\}du\,dv$$

$$=\tfrac{9}{4}\int_0^b[u^2(b^2-u^2)^3\sin^{-1}\sqrt{\frac{u}{b+u}}+u^5(bu-u^2)^{\frac{3}{2}}\log\left(\frac{\sqrt{(b-u)}+\sqrt{b}}{\sqrt{u}}\right)$$

$$+bu^2(b-u)^2(bu-b^2-u^2)\sqrt{(bu)}]du.$$

In the first term let $u=b\tan^2\theta$, in the second and third terms let $u=b\sin^2\theta$. Then